



Jitter Analysis Approach

August 21, 2003
Alice Liu (NASA GSFC Code 595)
Steve Andrews (NASA GSFC Code 595)



Introduction



- Solar Dynamic Observatory (SDO) mission has very tight pointing jitter requirements:
 - 5 arcsec (3 σ) between 0.02 and 50 Hz
 - 1 arcsec (3 σ) above 30 Hz
 - Wheel allocation is 2.55 arcsec (3 σ), TBR
- Objective of this analysis is to verify requirements can be met using preliminary finite element model and reaction wheel disturbance model
- Analytical results provide direct comparison between two different commercial wheels and their jitter performance
- Results will be reported to project to assist SDO wheel selection process



DOCS - Disturbance Analysis



- DOCS: Disturbances-Optics-Controls-Structures Framework for modeling and analysis of precision opto-mechanical space systems [H. Gutierrez et. al., MIT Space Systems Lab]
- DOCS toolbox developed by Dr. Carl Blaurock at MIDE Tech. is a MATLAB environment for efficiently solving dynamic problems - fast system performance evaluation and design parameter optimization (available to Goddard employees)
- Three types of disturbance analysis included in DOCS disturbance module:
 - Time domain: disturbance measured or modeled as function of time
 - Frequency domain: disturbance measured or modeled in power spectral densities (PSDs)
 - Lyapunov analysis: disturbance modeled as output of a shaping filter driven by white noise



Jitter Analysis Approach



 Measured reaction wheel disturbances, w, are propagated through the structural model to estimate the pointing jitter performance, z_{perf}.

W → Plant (G_p) Z_{perf}

 Performance spectral density matrix can be computed from disturbance spectral density matrix and system transfer function matrix:

$$S_{zz}(\mathbf{w}) = G_p(j\mathbf{w})S_{ww}(\mathbf{w})G_p^H(j\mathbf{w})$$

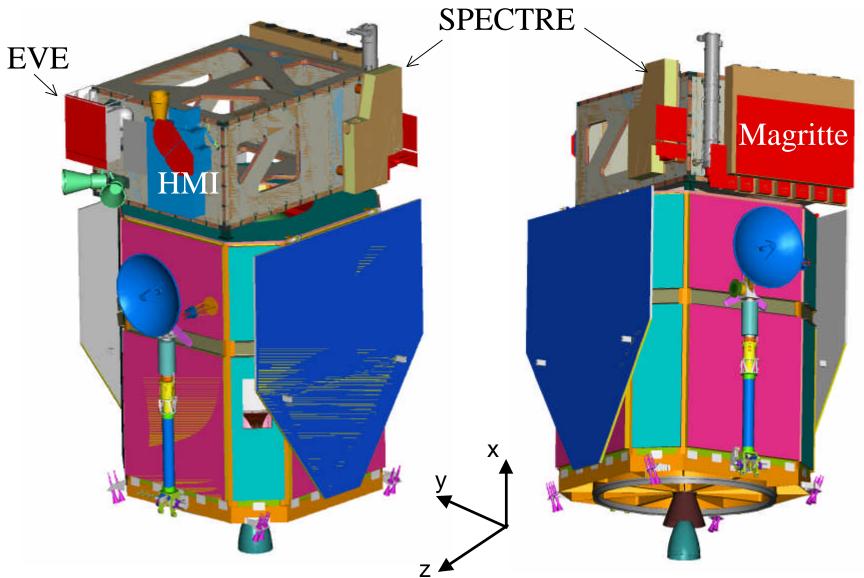
 Variance of ith performance metric (z_i) is the area under the power spectral density curve (assume zero-mean process)

$$\mathbf{S}_{z_i}^2 = \frac{1}{2\mathbf{p}} \int_{-\infty}^{+\infty} \left[S_{zz}(\mathbf{w}) \right]_{ii} d\mathbf{w} \approx \frac{1}{2\mathbf{p}} \int_{\mathbf{w}_{min}}^{\mathbf{w}_{max}} \left[S_{zz}(\mathbf{w}) \right]_{ii} d\mathbf{w}$$



SDO Observatory (Stowed Config.)







Structural Model



- Swales conducted finite element analysis and provided three models: beginning of life (BOL), 15% Fuel (no oxidizer), and end of life (EOL)
 - SDO deployed frequencies ω_r and normal modes ϕ_r to 100 Hz
 - Four reaction wheel input nodes and six optical instrument output nodes (Guide telescope, HMI, EVE, Magritte, Spectre, and KCOR)
- Create structural model from frequencies and normal modes

$$\begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix} \ddot{q} + \begin{bmatrix} \ddots & & \\ & 2\boldsymbol{z}_r \boldsymbol{w}_r & \\ & \ddots & \\ & & \ddots \end{bmatrix} \dot{q} + \begin{bmatrix} \ddots & & \\ & \boldsymbol{w}_r^2 & \\ & & \ddots & \\ & & & \ddots \end{bmatrix} \boldsymbol{q} = \boldsymbol{\Phi}^T \boldsymbol{b}_w \boldsymbol{w} \qquad \begin{aligned} & r = 1, \dots, \# \operatorname{modes} \\ & \boldsymbol{\Phi} & = [\boldsymbol{f}_1 & \dots & \boldsymbol{f}_{\# \operatorname{modes}}] \\ & \# \operatorname{dof} \times \# \operatorname{modes} \end{aligned}$$

$$r = 1, ..., \# \text{modes}$$

$$\Phi = [\mathbf{f}_1 \quad ... \quad \mathbf{f}_{\# \text{modes}}]$$

$$\# \text{dof} \times \# \text{modes}$$

State-space modal model

$$\begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -\Omega^2 & -2Z\Omega \end{bmatrix} \begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ \Phi^T \mathbf{b}_w \end{bmatrix} w$$

$$z_{perf} = \underbrace{\begin{bmatrix} C_{zp} \Phi & C_{zv} \Phi \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} q \\ \dot{q} \end{bmatrix}}_{D} + \underbrace{D_{zw}}_{0} w$$

$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$



$$\dot{x} = Ax + Bw$$

$$7x = Cx$$

Transfer function from w to z:

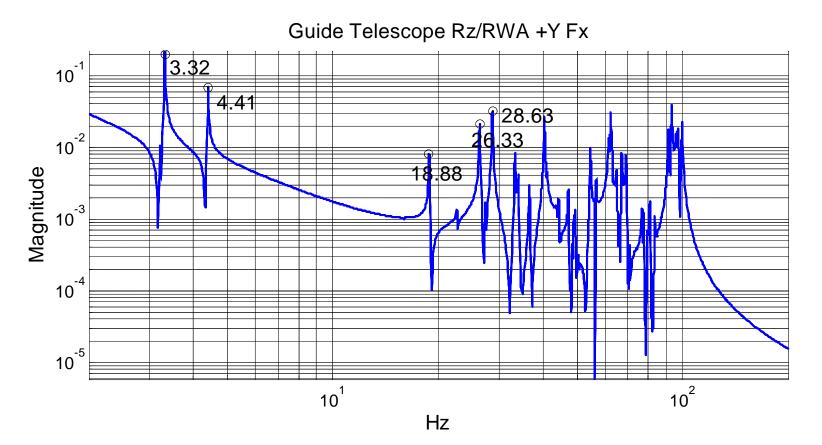
$$G_p(s) = C(sI - A)^{-1}B$$



Plant Transfer Function



- No optical model available output of the system is defined as the rotation around Y and Z axes (Ry, Rz) of instruments
- Assume translational motions and rotation around the lineof-sight will not affect pointing

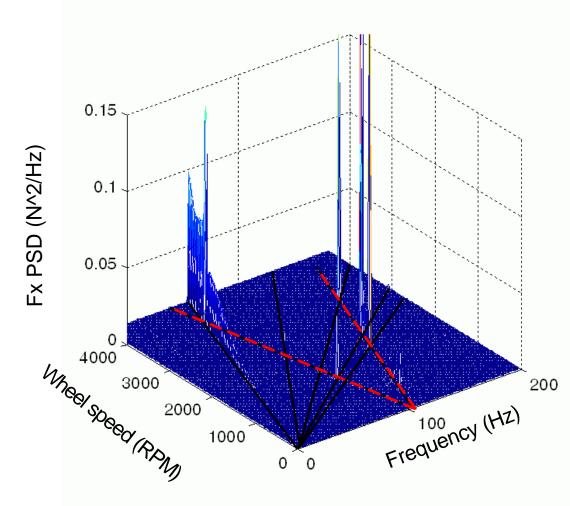




Reaction Wheel Disturbances



- Wheel force and torque disturbances are measured at steadystate wheel speeds
 - Data collected at 30 RPM increments from 0 to 6000 RPM
 - Fine balanced wheel
- PSD data available at discrete wheel speeds
- Data quality:
 - Low frequency data is inaccurate due to frequency resolution
 - High frequency data may be contaminated by test stand modes
 - Limit frequency range to
 2-200 Hz

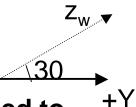




Disturbance Input



- Four wheels mounted on each face of the bus
 - Wheel spin axis is 30 degrees from the surface normal



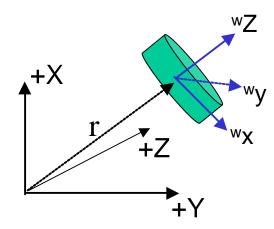
- Wheel induced force and torques must be transformed to spacecraft frame
 - Create rotation matrices (R) from local wheel frame to spacecraft body frame for each wheel
 - Account for additional moment caused by wheel disturbance force since origin of wheel frame does not coincide with spacecraft c.g.

$$\begin{bmatrix} s/c \\ t \end{bmatrix} = \begin{bmatrix} s/c R_w & 0 \\ S(r)^{s/c} R_w & s/c R_w \end{bmatrix}^w \begin{bmatrix} F \\ t \end{bmatrix} = s/c T_w \begin{bmatrix} F \\ t \end{bmatrix} + X$$

where S(r) is a skew - symmetric matrix

Transformed spectral density matrix:

$$S^{s/c}S_{ww} = S^{s/c}T_wS_{ww}S^{s/c}T_w^T$$

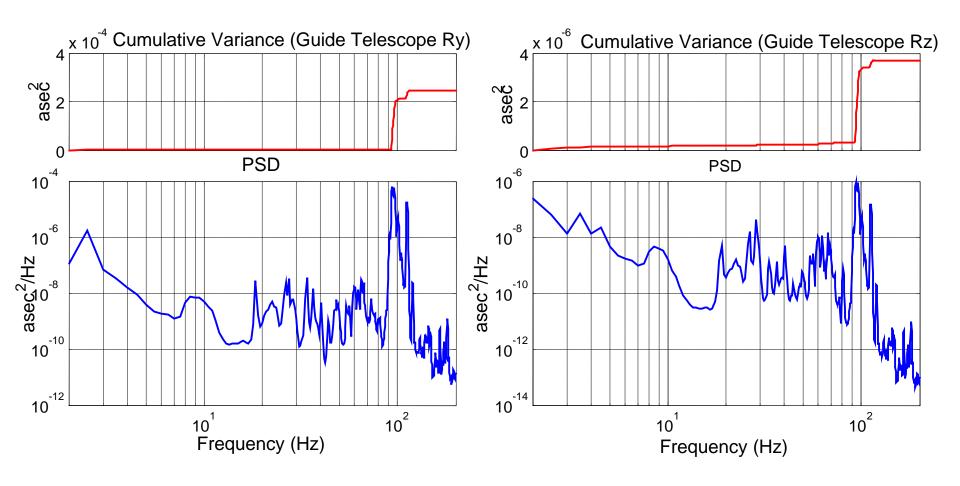




Jitter Analysis Output



 Jitter analysis outputs for Guide Telescope rotations around Y and Z axes with 4 wheels spinning at 500 RPM simultaneously





SDO Jitter Analysis



• RMS pointing of each instrument is the root-sum-square (RSS) of $s_{\rm Rv}$ and $s_{\rm Rz}$ at the corresponding structural node

$$\mathbf{s}_p = \sqrt{\mathbf{s}_{R_y}^2 + \mathbf{s}_{R_z}^2}$$

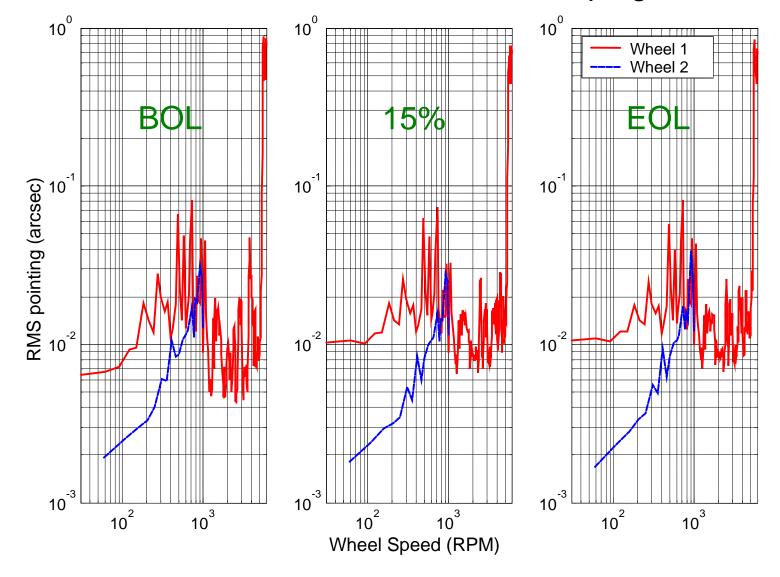
- RMS pointing is computed at each wheel speed, assuming quasi-steady-state operations
- All four wheels are assumed to spin at the same speed and impart similar disturbances on the spacecraft
- Two types of wheel disturbances are used for jitter analyses
- Total of six cases are examined for each instrument
 - BOL, 15% Fuel, and EOL
 - Uniform 0.2% and 0.5% damping



Guide Telescope Jitter results



Assume 0.2% uniform damping





Summary



- Preliminary wheel jitter analyses show that pointing jitter is less than 0.1 arcsec for reasonable wheel speed range
- Both candidate wheels are capable of meeting jitter requirements